

Complex

Trigonometric

* Roots of complex number :-

$$Z_K = (x+iy)^{\frac{1}{n}} = r^{\frac{1}{n}} e^{i \left(\frac{\theta + 2\pi K}{n} \right)}$$

* Find solution / Find value / Find roots

$$\boxed{1} \quad Z_K = (-2-2\sqrt{3}i)^{\frac{1}{4}} = r^{\frac{1}{4}} e^{i \left(\frac{\theta + 2\pi K}{n} \right)}$$

$$K=0,1,2,3$$

$$r = |Z| = \sqrt{x^2 + y^2} = \sqrt{4+12} = 4$$

$$\theta = \pi + \tan^{-1} \left(\frac{2\sqrt{3}}{2} \right) = \frac{4\pi}{3}$$

$$\underline{\underline{So}} \quad Z_K = (4)^{\frac{1}{4}} \cdot e^{i \left(\frac{\frac{4\pi}{3} + 2K\pi}{n} \right)}$$

$$Z_K = \sqrt{2} \left[\cos \left(\frac{\frac{4\pi}{3} + 2K\pi}{4} \right) + i \sin \left(\frac{\frac{4\pi}{3} + 2K\pi}{4} \right) \right]$$

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$$K=0 \Rightarrow Z_0 = \frac{1}{\sqrt{2}} + \frac{\sqrt{6}}{2} i$$

$$K=1 \Rightarrow Z_1$$

$$K_2 \Rightarrow Z_2 = \frac{-1}{\sqrt{2}} - \frac{\sqrt{6}}{2} i$$

$$K_3 \Rightarrow Z_3 = \frac{\sqrt{6}}{2} - \frac{1}{\sqrt{2}} i$$

*Find solution $Z^{\frac{3}{2}} = 4\sqrt{2} + i4\sqrt{2}$

$$\therefore Z_K = (4\sqrt{2} + i4\sqrt{2})^{\frac{2}{3}}$$

$$= r^{\frac{2}{3}} \cdot e^{i\left(\frac{\theta + 4\pi K}{3}\right)}$$

$$r = |Z| = \sqrt{x^2 + y^2} = \boxed{8}$$

$$\theta = \tan^{-1} \frac{4\sqrt{2}}{4\sqrt{2}} = \frac{\pi}{4}$$

$$\therefore Z_K = (8)^{\frac{2}{3}} \cdot e^{i\left(\frac{\frac{\pi}{4} + 4K\pi}{3}\right)}$$

$$\boxed{2}$$

$$Z_K = (8)^{\frac{2}{3}} \left[\cos\left(\frac{\frac{\pi}{4} + 4K\pi}{3}\right) + i \sin\left(\frac{\frac{\pi}{4} + 4K\pi}{3}\right) \right]$$

$$K=0 \Rightarrow Z_0 = 2\sqrt{3} + 2i$$

$$K=1 \Rightarrow Z_1 = -4i$$

$$K=2 \Rightarrow Z_2 = -2\sqrt{3} + 2i$$

$$* Z^3 - 1 = 0 \rightarrow Z^3 = 1 \rightarrow Z = (1)^{\frac{1}{3}}$$

$$Z_K = (1 + 0i)^{\frac{1}{3}} = r^{\frac{1}{3}} \cdot e^{i\left(\frac{\theta + 2K\pi}{3}\right)}$$

$$r = |Z| = \sqrt{x^2 + y^2} = \sqrt{1 + 0} = 1$$

$$\theta = \tan^{-1}\left(\frac{0}{1}\right) = 0$$

$$Z_K = 1 \cdot e^{i\left(\frac{2K\pi}{3}\right)} = \cos\left(\frac{2K\pi}{3}\right) + i \sin\left(\frac{2K\pi}{3}\right)$$

$$K=0 \rightarrow Z_0 = 1 + i0$$

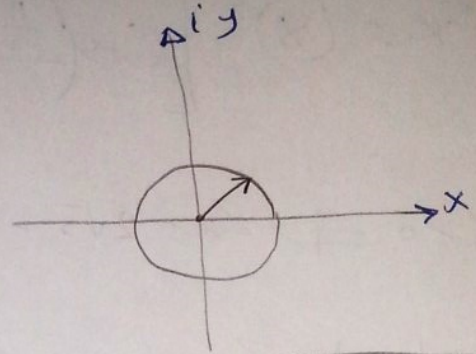
$$K=1 \rightarrow Z_1 = \frac{\sqrt{3}}{2} + i \frac{1}{2}$$

$$K=2 \rightarrow Z_2 = \frac{\sqrt{3}}{2} - i \frac{1}{2}$$

* Some Curves in Complex Plane :-

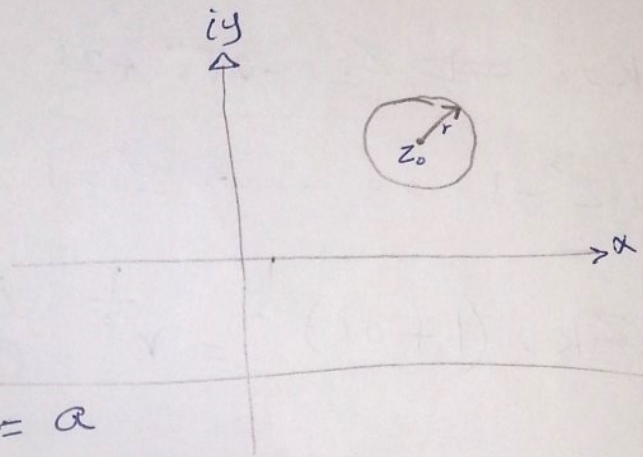
[1] $|z| = r$

← دائرة نصف قطرها r ومركزها نقطة الأصل.



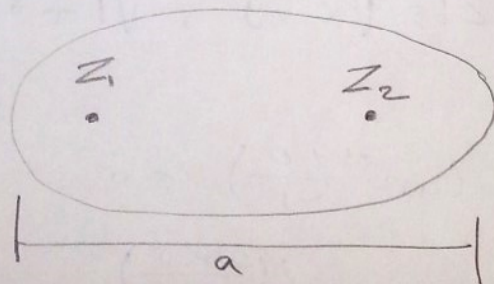
[2] $|z - z_0| = r$

← دائرة نصف قطرها r ومركزها z_0 .



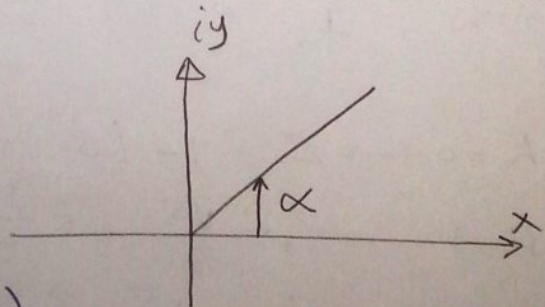
[3] $|z - z_1| + |z_1 - z_2| = a$

قطع ناقص



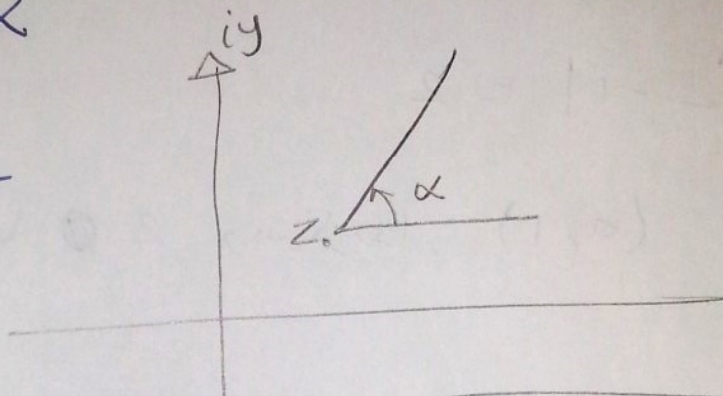
[4] $\arg(z) = \alpha$

← هي جميع النقاط الواقعة على الخط المستقيم الذي يمتنع زاوية (α) مع محور السينات ومنه نقطة الأصل.



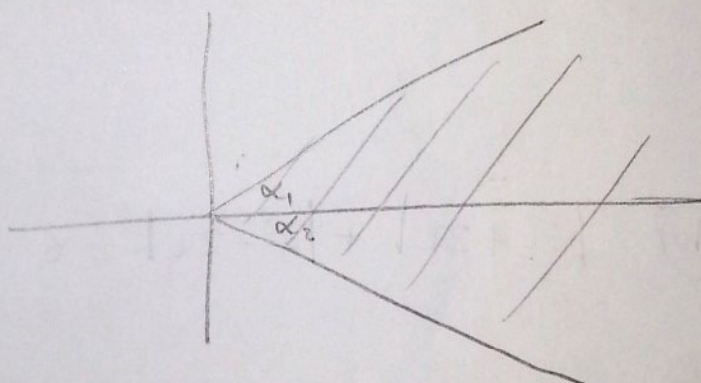
$$\boxed{5} \arg(z - z_0) = \alpha$$

← نفس الكلام ولكنه
من نقطة z_0 .



$$\boxed{6} |\arg(z)| \leq \alpha$$

$$-\alpha \leq \arg(z) \leq \alpha$$



هام

① لو $\alpha > \pi$ نرسم المنحنى فقط .

② المعادلة ليست كالمساوية نعوّف عن $(z = x + iy)$ ونشوف

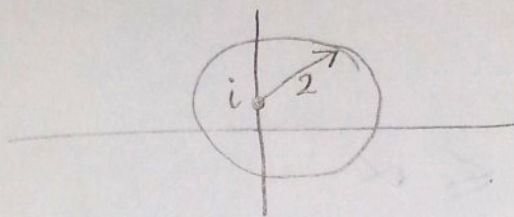
معادلة المنحنى الناتج تمثّل ليه ؟

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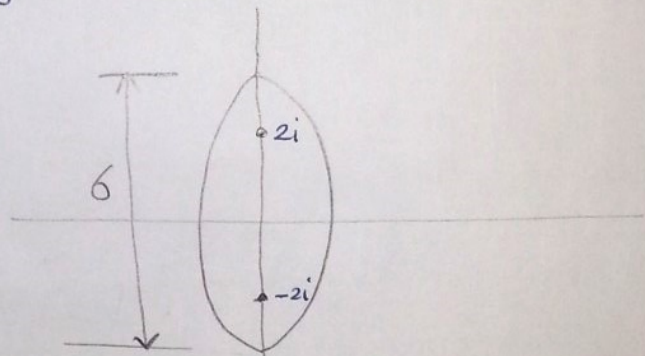
* Describe & graph

a) $|z - i| = 2$

هـ دائرة نصف قطرها 2 ومركزها (0, 1)



b) $|z + 2i| + |z - 2i| = 6$



c) $\left| \frac{z - i}{z + i} \right| = 1$

هـ ليست من الأشكال السابقة

~~xxxxxxxx~~
~~xxxxxxxx~~

~~xxxx~~
 $\frac{|z - i|}{|z + i|} = 1$

$$|x + iy - i| = |x + iy + i|$$

$$|x + i(y-1)| = |x + i(y+1)|$$

$$\sqrt{x^2 + (y-1)^2} = \sqrt{x^2 + (y+1)^2}$$

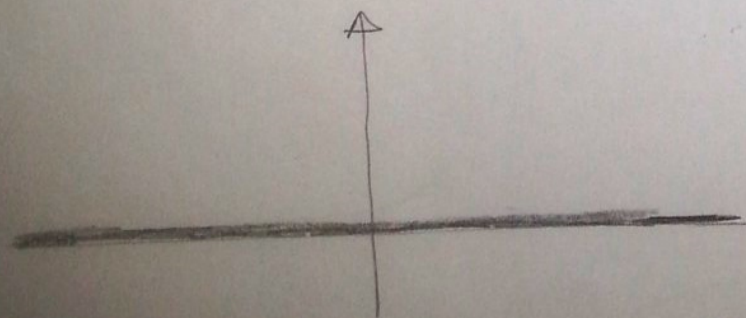
متر ببع الاكرومين

$$x^2 + (y-1)^2 = x^2 + (y+1)^2$$

$$y^2 - 2y + 1 = y^2 + 2y + 1$$

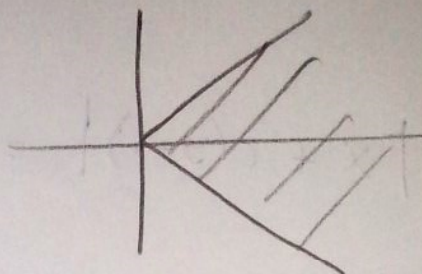
$$-y = y \rightarrow 2y = 0 \Rightarrow \boxed{y = 0}$$

محور السينات



$$\textcircled{d} |\arg(z)| \leq \frac{\pi}{4}$$

$$-\frac{\pi}{4} \leq \arg(z) \leq \frac{\pi}{4}$$



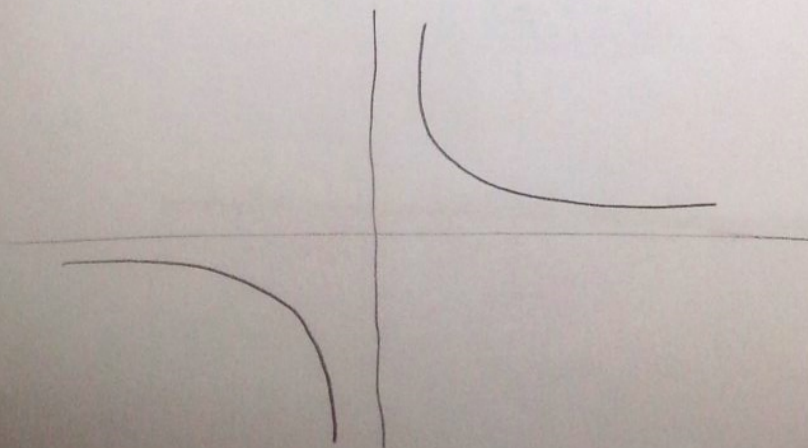
$$\textcircled{e} \operatorname{Im}(z^2) = 4$$

$$\operatorname{Im}[(x+iy)^2] = 4$$

$$\operatorname{Im}(x^2 + i2xy - y^2) = 4$$

$$2xy = 4 \Rightarrow xy = 2$$

$$y = \frac{2}{x}$$



* Write the following functions on the

Form $w = u + iv$

$$f(z) = u + iv$$

$(z = x + iy)$ نقطة
 $\ln(z)$, $z^{n/2}$ مؤثر
 $z = r e^{i\theta}$

$$\boxed{1} f(z) = z^2$$

$$= (x + iy)^2 = x^2 + i2xy - y^2$$

$$= \underbrace{(x^2 - y^2)}_u + i \underbrace{(2xy)}_v$$

$$\boxed{2} f(z) = \frac{1}{z} + z$$

$$= \frac{1 + (x + iy)(x + iy)}{x + iy} = \frac{1 + x^2 + i2xy - y^2}{x + iy}$$

$$x - iy$$

بالهز

$$= \frac{x + x^3 + xy^2}{x^2 + y^2} + i \frac{-y + y^3 + x^2y}{x^2 + y^2}$$

\downarrow \downarrow
 u v

99

$$\textcircled{3} f(z) = z \cdot e^{2z}$$

$$= (x+iy) \cdot e^{2(x+iy)}$$

$$= (x+iy) e^{2x} \cdot e^{2iy}$$

$$= e^{2x} (x+iy) (\cos(2y) + i \sin(2y))$$

$$= x e^{2x} \cos(2y) + i x e^{2x} \sin(2y) + i y e^{2x} \cos(2y) - y e^{2x} \sin(2y)$$

$$\textcircled{4} f(z) = \ln(z)$$

$$= \ln(re^{i\theta})$$

$$= \ln(r) + \ln e^{i\theta}$$

$$= \underbrace{\ln r}_u + \underbrace{i\theta}_v$$

* Differentiation

→ النتائج تعاد في المشتقة الأولى للمدالة

$$\lim_{\Delta z \rightarrow 0} \frac{f(z + \Delta z) - f(z)}{\Delta z} = f'(z)$$

$$\lim_y \lim_x = \lim_x \lim_y$$

example

* Show that $f(z) = 2x + ixy$ isn't diff.

$$\lim_{\Delta z \rightarrow 0} \frac{2(x + \Delta x) - i(x + \Delta x)(y + \Delta y) - 2x + ixy}{\Delta x + i\Delta y}$$

$$\lim_{\Delta z \rightarrow 0} \frac{2x + 2\Delta x - ixy - i\Delta xy - i\Delta x y - i\Delta x \Delta y - 2x + ixy}{\Delta x + i\Delta y}$$

$$\lim_{\Delta z \rightarrow 0} \frac{2\Delta x - i\Delta xy - i\Delta x y - i\Delta x \Delta y}{\Delta x + i\Delta y}$$

$$\lim_{\Delta y \rightarrow 0} \left[\lim_{\Delta x \rightarrow 0} \frac{2\Delta x - ix\Delta y - i\Delta xy - i\Delta x\Delta y}{\Delta x + i\Delta y} \right]$$

$$\lim_{\Delta y \rightarrow 0} \frac{-ix\Delta y}{i\Delta y} = \boxed{-x}$$

$$\lim_{\Delta x \rightarrow 0} \left[\lim_{\Delta y \rightarrow 0} \frac{2\Delta x - ix\Delta y - i\Delta xy - i\Delta x\Delta y}{\Delta x + i\Delta y} \right]$$

$$\lim_{\Delta x \rightarrow 0} \left[\frac{2\Delta x + i\Delta xy}{\Delta x} \right] = 2 - iy$$

$$\lim \neq \lim$$

↳ this function is not diff.

✗